Book Review

Transonic Aerodynamics Vol. 30 of North-Holland Series in Applied Mathematics and Mechanics, by J.D. Cole and L.P. Cook, Elsevier Science Publishers, 1986, 473 pp., $47.50

A masterpiece — an excellent review of the inviscid transonic small disturbance theory including the original contributions of the authors, their co-workers, and their students.

The material is presented in seven chapters. In the Introduction, the transonic regime is defined and typical transonic flow patterns are sketched. Next, the framework of the book is outlined; perturbation methods are used to give a systematic discussion of the first order theory.

The second chapter deals with the linearized theory and its breakdown in the transonic regime. Besides the acoustic equations, the exact (Euler) equations and the jump conditions admitted by their weak solution are discussed. Crocco's vorticity/entropy relation is used to justify the irrotationality assumption and the full potential model equation.

In Chapter 3, transonic expansion procedures for steady flow past airfoils and wings are outlined, first starting with the full potential equation and later starting with the Euler equations. The advantage of the second approach is freedom from assumptions of isentropy and irrotationality. (Indeed, the derivation is modified to account for a certain amount of vorticity in the upstream flow). Expansion procedures for jet flows, slender bodies (Area Rule) and unsteady flows are also discussed together with Transonic Similarity Rules as well as Lift and Drag Integrals for two- and three-dimensional lifting surfaces. Also in Chapter 3, hodograph equations for planar flows are derived with an interesting discussion of branch and limit lines.

Chapter 4 deals with Transonic Far Fields for sonic, supersonic, and subsonic free streams and for both two-dimensional and axisymmetric flows. Lifting three-dimensional wings and unsteady two-dimensional flows with subsonic free streams are also included.

Chapter 5 is devoted to Transonic Airfoil Theory. After the problem is formulated the nose singularity and the flow in the neighborhood of the intersection of a shock wave and a curved surface are analyzed. Next, numerical methods for plane steady flows are studied in detail, including conservative type dependent difference schemes, line relaxation algorithms, calculations of wave drag and comparison of integrals along the shock and the body as well as applications to the tunnel-wall interference problem. Also in Chapter 5, the hodograph equations are used to calculate the sonic flow around realistic airfoil shapes. This chapter is concluded with a discussion of the Stabilization Law, where the equations for the correction to sonic flows due to variations in the Mach number are derived.

In Chapter 6, the Transonic Lifting Line Theory for unswept and swept wings is studied for both subsonic and sonic free streams. The equations for the finite aspect ratio correction, at each spanwise station, are derived. It is shown that for a special case of a wing with similar sections, the boundary value problem for the correction can be scaled to be independent of the spanwise coordinate. The equation is linear but its coefficients depend on the solution of the nonlinear two-dimensional problem, hence numerical methods are needed. A proof of the uniqueness of the first order correction for subsonic free streams, in the case that the cross-sectional flow is shock free, is promised, but the details are not given.

Finally, in Chapter 7, supersonic wings whose leading edges are swept close to the Mach angle, are analyzed. First, the linearized supersonic theory is presented, followed by the derivation of the nonlinear quasi transonic equations. Application to a delta wing with wedge cross-section is discussed including an estimate of the drag correction due to the quasi transonic effects based on a numerical solution of the reduced elliptic problem in terms of conical cylindrical coordinates.

Throughout the book, singular perturbation problems are identified and inner and outer matched asymptotic expansions are systematically developed. The consistency of the representation together with the authors' style of combining mathematical and physical arguments in their discussions works remarkably well and convinces the reader of the authors' point of view. For example, in Chapter 3 the transonic expansion procedure is applied to the potential equation in nonconservative (quasi linear) form. The resulting first order equation is then recast in conservation form. The jump condition admitted by its weak solution is checked to be consistent with the Rankine-Hugoniot relations within the small disturbance approximations. An alternative point of view is to start with the conservative potential equation and to preserve the conservation form through the expansion procedure. The two approaches may not lead to the same equation, even for first order approximations, as in the particular example of three-dimensional flows around swept wings. Although the two equations are valid approximations, their numerical solutions are different. Also flux boundary conditions may be imposed leading to a completely conservative formulation. These are simple fixes, without resorting to formal second order approximations. In fact, there are some technical difficulties with the latter, particularly at the sonic points.

It is obvious that the book has concentrated mainly on the developments made by the authors and their co-workers. In this respect and as far as the topics covered in the book, the authors were very successful. There is, however, more to "transonic aerodynamics" than the inviscid small disturbance theory and even for that, only closely related works are referenced in the text. It is clear that some references are missing; even some books on transonic flows are not mentioned.

This observation particularly holds for the part on numerical methods, which is mainly based on the breakthrough of Murman and Cole and the extensions by Murman. For example, contributions from NASA Ames and Langley Research Centers are not mentioned (except for Klunker's far-field development). More seriously, Murman's scheme admits expansion shocks, violating the entropy condition. Other schemes, which produce only compression shocks, are not mentioned.

Moreover, on page 38, after the derivation of the potential equation, it is stated, "With this statement of the problem the solution presumably exists and is unique." Except for a footnote on page 279 ("Recent calculations have indicated the possibility of nonuniqueness for a narrow range of subsonic Mach numbers"), the nonuniqueness problem is not addressed.

Related to this point is the work on the nonisentropic potential formulation; again it is not considered. Strictly speaking, distributed vorticity is introduced downstream of shockwaves; the only exceptions are shocks of uniform strength such as plane or conical shocks. It seems, however, that for some flows, where the shock curvature is small, the vorticity generated can be neglected while the entropy jump may be accounted for. Such a modification to the potential formulation could be important!

One may argue, however, that the scope of the book is focused on the analytical and not the computational aspects of transonic flows. But within the inviscid transonic small disturbance theory, the following topics are missing:

a) The authors concentrate on external flows; internal flows and wind tunnel wall interference problems are omitted.
b) While steady flows are thoroughly studied, unsteady flows are lightly touched.
c) Only analysis problems are discussed; design methods are omitted.
d) Among the analysis problems of steady, external flows, the authors treated only high aspect ratio wings; the lifting low aspect ratio wings are not covered.

One remedy to all this omission is simply to consider the book as special topics or collected works, by the authors. In fact, Professor Cole has been a leader of this field since his pioneering work in the late 1940s and this book is an outcome of "Forty Years of Transonic Flow."

At any rate, the book should be judged by what it contains, not by what is missing. Indeed the book contains important contributions by the authors and it will be extremely useful for anyone who is interested in inviscid transonic small disturbance theory and in singular perturbation and matched asymptotic expansion for many years to come. It is also highly recommended for anyone who is interested in transonic aerodynamics in general. This book is good for both students and specialists and should be in every library of applied mathematics and mechanics.

M. Hafez
University of California, Davis